ECE 536 – Spring 2022

Homework #3 – Solutions

Problem 1)

(A) SINGLE MODE REGIME Remember that the first mode has no cutoff, and the second mode is odd, and begins when $k_x d/2 = \pi/2$. Therefore, we can write

$$R_{max} = \left(\frac{2\pi}{\lambda_0}\right)\frac{d_0}{2}\sqrt{n_1^2 - n^2} = \frac{\pi}{2} \tag{1.1}$$

to see that $d_0 = 441\text{nm}$.

(B) GRAPHICAL APPROACH This is the same approach as the quantum well problem, except we now have

$$\begin{aligned} R &= \frac{2\pi}{\lambda_0}\frac{d}{2}\sqrt{n_1^2 - n^2} \\ \alpha\frac{d}{2} &= k_x\frac{d}{2}\tan\left(k_x\frac{d}{2}\right) \end{aligned} \tag{1.2}$$

We define $x = k_x d/2$ and $y = \alpha d/2$ and find the intersection points of the two lines. The plot is located in Fig. 1.1. Finding the intersection points gives $\alpha = 5.548\mu m^{-1}$, $k_x = 4.466\mu m^{-1}$, and $k_z = 15.37\mu m^{-1}$.

The plot of the waveguide mode is shown in Fig. 1.2.

(C) CONFINEMENT FACTOR To find the confinement factor, we need to find the ratio of power confined inside the waveguide to the power in the optical mode. Or, mathematically,

$$\Gamma = \frac{\frac{1}{2}\int_{inside}\Re\left(\vec{E}\times\vec{H}^*\right)\cdot\hat{z}dx}{\frac{1}{2}\int_{total}\Re\left(\vec{E}\times\vec{H}^*\right)\cdot\hat{z}dx} \tag{1.3}$$

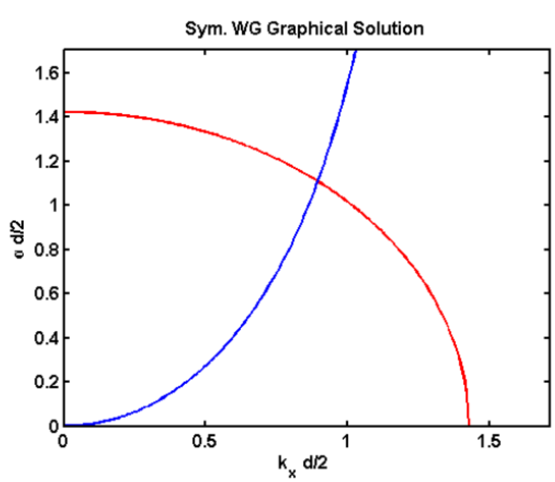

Figure 1.1: Graphical method of solving 1-D dielectric slab waveguide.

In our case, because the waveguide is symmetric and the mode is the fundamental TE mode, we can simplify this to

$$\Gamma = \frac{\int_{inside} \frac{1}{n_1} \left|E_y\right|^2 dx}{\int_{total} \frac{1}{n_i} \left|E_y\right|^2 dx} \tag{1.4}$$

which gives 0.7968, meaning only 80% of the power contained in the mode is located within the waveguide because it is so small.

If the In mole fraction of the cladding layer is raised, then the index of refraction of the cladding layer will increase, thereby shrinking the contrast between the waveguide and the cladding. Thus, the optical confinement will decrease since more of the mode will "leak" into the cladding.

Problem 2)

The two physical problems share a similarity in both the form of the differential equation and the mode (eigen) solutions. Because a basic tenet of quantum mechanics is that the electron acts both as a particle and as a wave, this is intuitive. The quantum mechanical potential V_0 is analogous to the optical index of refraction n, as both serve to confine the mode (or wave function). A list of the requested parameters is given below:

Governing Equations

$$QW: \begin{cases} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + V(x)\right] \phi(x) e^{ik_z z} = E\phi(x) e^{ik_z z} \\ \left[\frac{\partial^2}{\partial x^2} - k_z^2 + V(x) - E\right] = 0 \end{cases}$$

$$WG: \begin{cases} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu \epsilon(x)\right] E_y(x) e^{ik_z z} = 0 \\ \left[\frac{\partial^2}{\partial x^2} - k_z^2 + \omega^2 \mu \epsilon(x)\right] E_y(x) = 0 \end{cases}$$

Wavefunction/Mode

$$QW: \begin{array}{l} \phi(x) = C_1 \cos\left(k_x \frac{d}{2}\right) e^{-\alpha(|x| - d/2)} \\ \phi(x) = C_1 \cos(k_x x) \end{array}$$

$$WG: \begin{array}{l} E_y(x) = C_1 \cos\left(k_x \frac{d}{2}\right) e^{-\alpha(|x| - d/2)} \\ E_y(x) = C_1 \cos(k_x x) \end{array}$$

Propagation/Decay Constants

$$QW: \begin{array}{l} -\alpha^2 + k_z^2 = \frac{2m^*}{\hbar^2}(E - V_0) \\ k_x^2 + k_z^2 = \frac{2m_1^*}{\hbar^2} E \end{array}$$

$$WG: \begin{array}{l} -\alpha^2 + k_z^2 = \omega^2 \mu \epsilon \\ k_x^2 + k_z^2 = \omega^2 \mu_1 \epsilon_1 \end{array}$$

Guidance Conditions

$$QW: \begin{array}{l} \alpha\frac{d}{2} = \frac{m^*}{m_1^*} k_x \frac{d}{2} \tan\left(k_x \frac{d}{2}\right) \\ \left(k_x \frac{d}{2}\right)^2 + \frac{m^*}{m_1^*}\left(\alpha\frac{d}{2}\right)^2 = 2\frac{m_1^* V_0}{\hbar^2}\left(\frac{d}{2}\right)^2 \end{array}$$

$$WG: \begin{array}{l} \alpha\frac{d}{2} = \frac{\mu}{\mu_1} k_x \frac{d}{2} \tan\left(k_x \frac{d}{2}\right) \\ \left(k_x \frac{d}{2}\right)^2 + \left(\alpha\frac{d}{2}\right)^2 = \omega^2\left(\mu_1\epsilon_1 - \mu\epsilon\right)\left(\frac{d}{2}\right)^2 \end{array}$$

Problem 3)

(A) Index of AlGaAs The index of refraction is not trivially related to the band gap of an alloy. The equations on p. 264 of the text list the equations for calculating the index of refraction for $Al_xGa_{1-x}As$ for a given mole fraction x and photon energy $\hbar\omega$. In practice, when designing a waveguide, it is best to find the index for a large range of mole fractions and energies. However, in this problem, we are looking to find the index for a handful of mole fraction values.

Therefore, for this problem, we will consider an iterative process given a single mole fraction and an array of energies (less than the band gap only, where the approximations given on p. 264 are accurate). First, find the band gap with the given mole fraction, then create an array of energies from a nominal value (1 eV, for example) up to the band gap. Find the band parameters Δ, A, and B for the given mole fraction. Then, find y and y_{so} for the whole range of photon energies being considered. And, finally, find an array for the permittivity and then the index. Fig. 3.1 shows the results of this process.

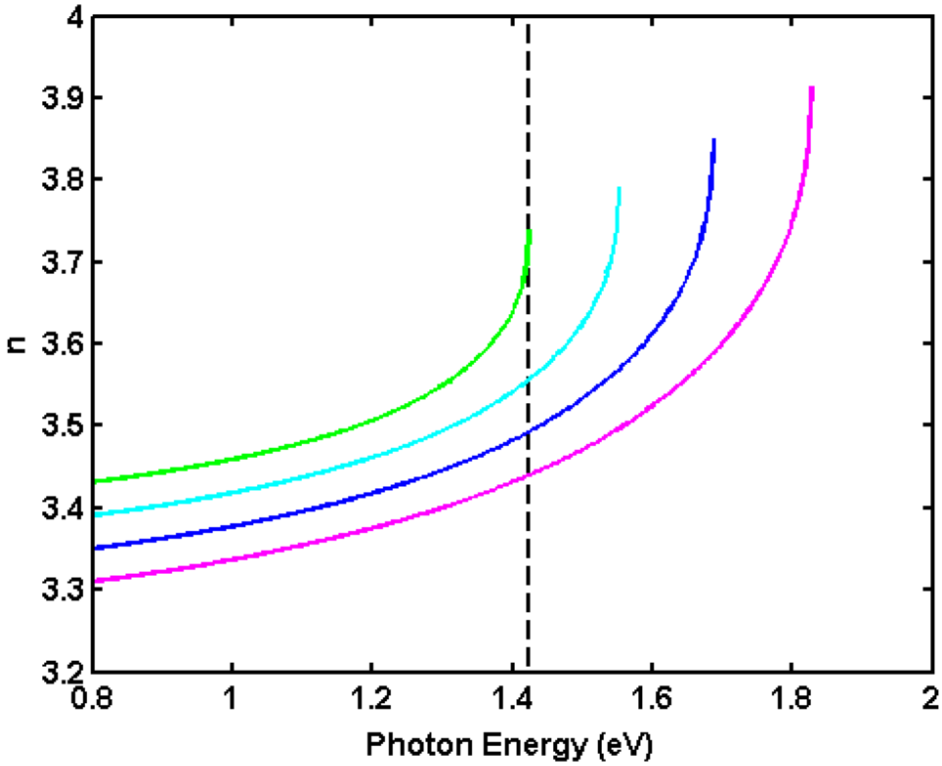

Figure 3.1: Index of refraction n as a function of photon energy for a variety of $Al_xGa_{1-x}As$ mole fractions.

(B) DESIGN Now, we can use the indices of refraction to design a waveguide. The core will be GaAs, which according to the plot of index vs. mole fraction, has an index of 3.739 near its band gap of 1.424 eV (shown on the plot by the black dashed line). For $x = 0.1$, the index is 3.556, $x = 0.2$, the index is 3.493, and for $x = 0.3$, the index is 3.440. The maximum single mode thicknesses are then 0.377 μm, 0.326 μm, 0.297 μm respectively.

It is important to note here that although larger contrast gives better mode confinement, it also requires a smaller thickness to ensure single mode guidance.

Problem 4)

(A) MODE COUNTING Start by finding the geometrical R-value of the problem

$$R = k_0 \frac{d}{2} \sqrt{n_1^2 - n^2} = 1.89\pi = 5.94 \tag{4.1}$$

Since each mode is cutoff at a multiple of $\pi/2$, we need to find $R < N\pi/2$, so N=4 here, meaning that the TE_0, TE_1, TE_2, and TE_3 modes are guided.

(B) PROPAGATION ANGLE This problem requires finding first the transverse propagation constant for each mode (k_x). This is done in the same way as Problem 1 above. The graphical solution is shown in Fig. 4.1. To find the propagation angle, then, we simply use the relation

$$\theta_m = \cos^{-1} \frac{k_{1x}}{k_1} \tag{4.2}$$

where k_1 is simply the k-number inside the waveguide ($14.66 \mu m^{-1}$). This gives propagation angles of 84.74°, 79.49°, 74.25°, and 69.17° for TE_{0-3}, respectively.

(C) CUTOFF PROPERTIES First, for the decay constant α at cutoff, it is always 0. That is, for any guided mode that exactly meets the cutoff condition, the decay constant outside the waveguide is identically zero. For the cutoff wavelength, we can directly rearrange the cutoff condition expression to see

$$R = \frac{2\pi}{\lambda_m} \frac{d}{2} \sqrt{n_1^2 - n^2} = m \frac{pi}{2} \rightarrow \lambda_m = \frac{2d\sqrt{n_1^2 - n^2}}{m} \rightarrow \lambda_m = \frac{5.67 \mu m}{m} \tag{4.3}$$

which gives cutoff wavelengths λ_0 =no cutoff, $\lambda_1 = 5.67 \mu m$, $\lambda_2 = 2.835 \mu m$, $\lambda_3 = 1.89 \mu m$. To find the propagation constant at cutoff, we simply have

$$k_{zm} = \frac{2\pi}{\lambda_m} n \tag{4.4}$$

which gives $k_z = 3.546, 7.09, 10.63 \mu m^{-1}$ for $TE_{1,2,3}$ respectively.

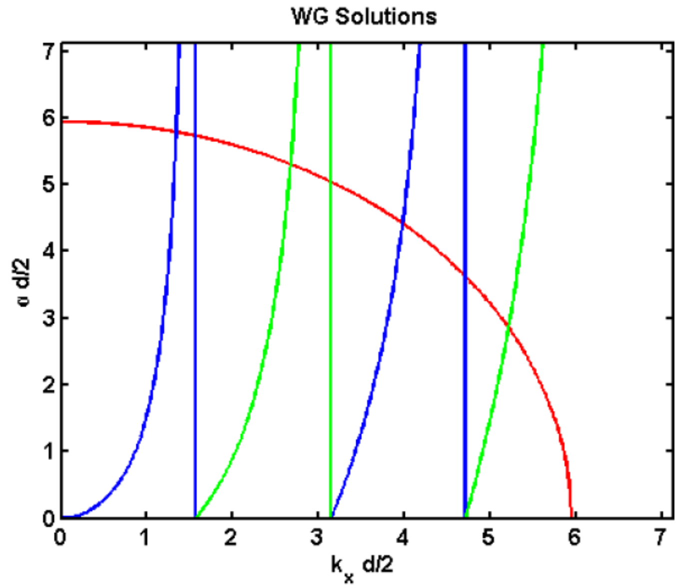

Figure 4.1: Graphical solution for symmetric slab waveguide.